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# Weak-localization corrections to the conductivity of double quantum wells

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**Abstract.** The weak-localization contribution $\delta\sigma(B)$ to the conductivity of a tunnel-coupled double-layer electron system is evaluated and its behaviour in weak magnetic fields $B$ perpendicular or parallel to the layers is examined. In a perpendicular field $B$, $\delta\sigma(B)$ increases and remains dependent on the tunnelling as long as the magnetic field is smaller than $\hbar/eD\tau_t$, where $D$ is the in-plane diffusion coefficient and $\tau_t$ the interlayer tunnelling time. If $\tau_t$ is smaller than the inelastic scattering time, a parallel magnetic field also leads to a considerable increase of the conductivity starting with a $B^2$-law and saturating at fields higher than $\hbar/eZ\sqrt{D\tau_t}$, where $Z$ is the interlayer distance. In the limit of coherent tunnelling, when $\tau_t$ is comparable to the elastic scattering time, $\delta\sigma(B)$ differs from that of a single-layer system due to ensuing modifications of the diffusion coefficient. A possibility for probing the weak-localization effect in double-layer systems by means of the dependence of the conductivity on the gate-controlled level splitting is discussed.

## 1. Introduction

The interference of electronic waves leads to negative corrections to the conductivity of electron systems. This effect is known as weak localization. It can be observed at very low temperatures, when the inelastic scattering rate is so small that the phase coherence is kept over many acts of elastic scattering. Although these quantum corrections are usually small, they can be distinguished due to their specific dependence on temperature, magnetic field, and frequency of the applied electric field.

The fundamentals of the theory of weak localization have been developed in references [1–4] and a review is given in reference [5]. Recently considerable attention [6–16] has been drawn to weak-localization phenomena in layered systems, where characteristic features are caused by a dimensionality crossover, as well as in single-layer systems [17]. These studies have been applied mostly to multilayer systems (superlattices) although the cases of barrier-separated thin metallic films [9, 10] and single-barrier structures [12] have also been investigated. In this paper we calculate the weak-localization corrections to the in-plane conductivity of two tunnel-coupled two-dimensional (2D) layers, the system which is typically formed in double-quantum-well structures [18]. Such double-layer electron systems represent an intermediate case between a single 2D layer and a three-dimensional superlattice and their various interesting properties are caused mainly by this fact. As regards the weak-localization

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problem, the tunnel coupling introduces an additional cut-off parameter in the diffusion pole, which implies that the weak-localization contribution to the conductivity depends on the strength of this coupling. This happens when the probability of tunnelling is not small in comparison to that of inelastic scattering. Another important consequence of the tunnel coupling is that a weak magnetic field $B$ applied *parallel* to the layers leads to a delocalization as does a field $B$ applied *perpendicular* to the layers [4]. The physical origin of the effect of the parallel field is explained in a similar way: this field introduces an additional phase for the electron which moves in one layer, then tunnels into another layer, moves there, and finally tunnels back and returns to the initial point. The phenomena described are the subject of the present study.

The paper is organized as follows. In section 2 we present the basic formalism for the calculation of the weak-localization contribution to the conductivity of a double-layer system. In section 3 we calculate the magnetoconductivity in a *perpendicular* magnetic field $B$ and in section 4 we repeat the calculation for a *parallel* field $B$. In section 5 we study the magneto-conductivity in conditions of coherent tunnel coupling, when the tunnelling probability is comparable to or greater than the probability of elastic scattering. In section 6 we discuss the results and consider the possibility of probing the weak-localization effects in double quantum wells by examining the dependence of the conductivity on the gate-controlled energy splitting of the lowest two levels of the double quantum well.

## 2. Formalism

The Hamiltonian of the double-quantum-well system is given, in the basis of the left ($l$) and right ($r$) layer orbitals $|l\rangle$ and $|r\rangle$, by

$$\hat{H}(\boldsymbol{x}) = \hat{P}_l\,[E_l(\boldsymbol{x}) + V_l(\boldsymbol{x})] + \hat{P}_r\,[E_r(\boldsymbol{x}) + V_r(\boldsymbol{x})] + \hat{h} \tag{1}$$

where $\boldsymbol{x} = (x, y)$ is the in-plane position vector; $E_j(\boldsymbol{x}) = \langle j|(-\mathrm{i}\hbar\,\partial/\partial\boldsymbol{x} + e\boldsymbol{A}(\boldsymbol{x}, z))^2|j\rangle/2m$ ($j = l, r$) is the matrix element of the kinetic energy operator, $\boldsymbol{A}(\boldsymbol{x}, z)$ the vector potential, $e$ the elementary charge, $m$ the effective mass, and $V_j(\boldsymbol{x}) = \langle j|V(\boldsymbol{x}, z)|j\rangle$ the matrix element of the disorder potential. Furthermore,

$$\hat{h} = \frac{\Delta}{2}\hat{\sigma}_z + T\hat{\sigma}_x \tag{2}$$

is the potential energy matrix of the double-quantum-well system expressed through the energy level splitting $\Delta$ and tunnelling matrix element $T$. Finally, $\hat{\sigma}_i$ are the Pauli matrices and $\hat{P}_l = (1 + \hat{\sigma}_z)/2$ and $\hat{P}_r = (1 - \hat{\sigma}_z)/2$ the projection matrices.

According to the Kubo formula, the zero-temperature dc conductivity is given by

$$\sigma_{\alpha\alpha'} = \frac{e^2\hbar}{2\pi m^2 L^2}\sum_{bb'=R,A}(-1)^l\,\mathrm{Tr}\int\int \mathrm{d}\boldsymbol{x}\,\mathrm{d}\boldsymbol{x}'\,\left\langle \hat{\pi}^{\alpha}(\boldsymbol{x})\hat{G}^{b}_{E_F}(\boldsymbol{x}, \boldsymbol{x}')\hat{\pi}^{\alpha'}(\boldsymbol{x}')\hat{G}^{b'}_{E_F}(\boldsymbol{x}', \boldsymbol{x})\right\rangle \tag{3}$$

where $\hat{G}^{R,A}_{E_F}(\boldsymbol{x}, \boldsymbol{x}')$ are the Green's functions of the electron system with Fermi energy $E_F$, $L^2$ is the normalization area, $\hat{\pi}^{\alpha}(\boldsymbol{x})$ is the kinematic momentum operator, whose matrix elements are given as $[\hat{\boldsymbol{\pi}}(\boldsymbol{x})]_{jj} = \langle j| - \mathrm{i}\hbar\,\partial/\partial\boldsymbol{x} + e\boldsymbol{A}(\boldsymbol{x}, z)|j\rangle$, and $\alpha$ and $\alpha'$ are the coordinate indices $(x, y)$. The angular brackets $\langle\cdots\rangle$ denote statistical averaging, 'Tr' denotes the trace, $l = 1$ for $b = b'$ and $l = 0$ for $b \neq b'$.

The weak-localization contribution $\delta\sigma_{\alpha\alpha'}$ to the in-plane conductivity is given by the sum of an infinite set of diagrams with two or more maximally crossed impurity lines. Below, we consider a system of randomly distributed elastic scatterers (impurities) described by the correlation function $\langle V_j(\boldsymbol{x})V_{j'}(\boldsymbol{x}')\rangle = W_{jj'}(\boldsymbol{x},\boldsymbol{x}') = w_j\delta_{jj'}\delta(\boldsymbol{x}-\boldsymbol{x}')$. We obtain

$$\delta\sigma_{\alpha\alpha'} = \frac{e^2\hbar}{2\pi m^2 L^2}\sum_{bb'=R,A}(-1)^l \sum_{jj'j_1j_2}\int\int \mathrm{d}\boldsymbol{x}\,\mathrm{d}\boldsymbol{x}'\,\pi^{\alpha}_{jj}(\boldsymbol{x})G^b_{jj_1}(\boldsymbol{x},\boldsymbol{x}_1)G^b_{j_2j'}(\boldsymbol{x}_2,\boldsymbol{x}') \\ \times \pi^{\alpha'}_{j'j'}(\boldsymbol{x}')G^{b'}_{j'j_1}(\boldsymbol{x}',\boldsymbol{x}_1)G^{b'}_{j_2j}(\boldsymbol{x}_2,\boldsymbol{x})\tilde{C}^{bb'}_{j_1j_2}(\boldsymbol{x}_1,\boldsymbol{x}_2) \tag{4}$$

where $G^b_{jj'}(\boldsymbol{x},\boldsymbol{x}') = \langle[\hat{G}^b_{E_F}(\boldsymbol{x},\boldsymbol{x}')]_{jj'}\rangle$ are the averaged one-particle Green's functions, and $\tilde{C}^{bb'}_{j_1j_2}(\boldsymbol{x}_1,\boldsymbol{x}_2)$ is the Cooperon, the solution of the Bethe–Salpeter equation, given by

$$\tilde{C}^{bb'}_{j_1j_2}(\boldsymbol{x}_1,\boldsymbol{x}_2) = w_{j_1}G^b_{j_1j_2}(\boldsymbol{x}_1,\boldsymbol{x}_2)G^{b'}_{j_1j_2}(\boldsymbol{x}_1,\boldsymbol{x}_2)w_{j_2} \\ + w_{j_1}\sum_j\int \mathrm{d}\boldsymbol{x}\, G^b_{j_1j}(\boldsymbol{x}_1,\boldsymbol{x})G^{b'}_{j_1j}(\boldsymbol{x}_1,\boldsymbol{x})\tilde{C}^{bb'}_{jj_2}(\boldsymbol{x},\boldsymbol{x}_2). \tag{5}$$

In the following we use equations (4) and (5) to calculate $\delta\sigma_{\alpha\alpha'}$ for both directions of the magnetic field, perpendicular and parallel to the layers.

## 3. Perpendicular magnetic field

Consider the case when the field $B$ is directed perpendicular to the layers, $\boldsymbol{B} = (0, 0, B)$. In the Landau gauge, $\boldsymbol{A} = (0, Bx, 0)$, we can write the Green's function as

$$G^b_{j_1j_2}(\boldsymbol{x}_1,\boldsymbol{x}_2) = \frac{1}{2\pi\ell^2}\mathrm{e}^{-\mathrm{i}(x_1+x_2)(y_1-y_2)/2\ell^2}\ \mathrm{e}^{-(\boldsymbol{x}_1-\boldsymbol{x}_2)^2/4\ell^2}\sum_n L^0_n((\boldsymbol{x}_1-\boldsymbol{x}_2)^2/2\ell^2)G^b_{j_1j_2}(n) \tag{6}$$

where $G^b_{j_1j_2}(n)$ is the averaged Green's function in the Landau-level representation, $L^0_n$ are the Laguerre polynomials, and $\ell = \sqrt{\hbar/eB}$ is the magnetic length. From equations (5) and (6) it follows that the Cooperon may be written as

$$\tilde{C}^{bb'}_{j_1j_2}(\boldsymbol{x}_1,\boldsymbol{x}_2) = \mathrm{e}^{-\mathrm{i}(x_1+x_2)(y_1-y_2)/\ell^2}C^{bb'}_{j_1j_2}(|\boldsymbol{x}_1-\boldsymbol{x}_2|) \tag{7}$$

where $C^{bb'}_{j_1j_2}$ is its translationally invariant part, which is also called a Cooperon in the following. Introducing $C^{bb'}_{jj'}(p)$ as the Fourier transform of $C^{bb'}_{jj'}(x)$, we obtain

$$C^{bb'}_{j_1j_2}(p) = w_{j_1}\Lambda^{bb'}_{j_1j_2}(p)w_{j_2} + w_{j_1}\sum_{p',j}\int \mathrm{d}\boldsymbol{x}\,\mathrm{e}^{(\mathrm{i}/\hbar)(\boldsymbol{p}'-\boldsymbol{p})\cdot\boldsymbol{x}} \\ \times \Lambda^{bb'}_{j_1j}([(p'_x-\hbar y/\ell^2)^2+(p'_y+\hbar x/\ell^2)^2]^{1/2})C^{bb'}_{jj_2}(p') \tag{8}$$

where $\Lambda^{bb'}_{j_1j_2}(p)$ is the Fourier transform of the translationally invariant part of the product $G^b_{j_1j_2}(\boldsymbol{x}_1,\boldsymbol{x}_2)G^{b'}_{j_1j_2}(\boldsymbol{x}_1,\boldsymbol{x}_2)$. In the Landau-level representation it can be expressed as

$$\Lambda^{bb'}_{jj'}(p) = \frac{1}{2\pi\ell^2}\sum_{nn'}(-1)^{n+n'}\exp(-u)L^{n-n'}_n(u)L^{n'-n}_{n'}(u)G^b_{jj'}(n)G^{b'}_{jj'}(n') \tag{9}$$

where $u = p^2\ell^2/2\hbar^2$. Below, we consider very weak fields $B$, $\ell^2 \gg (v_{Fj}\tau_j)^2$, where $v_{Fj}$ and $\tau_j$ are, respectively, the Fermi velocities and elastic scattering times in the layers. In this *diffusion limit* we can expand $\Lambda^{bb'}_{j_1j}(\cdots)$ of equation (8) in series in $\hbar x/\ell^2$ and $\hbar y/\ell^2$. From equation (8) we obtain a set of differential equations

$$C^{bb'}_{j_1j_2}(p) = w_{j_1}\Lambda^{bb'}_{j_1j_2}(p)w_{j_2} + w_{j_1}\sum_j\left[\Lambda^{bb'}_{j_1j}(p)C^{bb'}_{jj_2}(p) - \frac{\hbar^4}{2\ell^4}\frac{\mathrm{d}^2\Lambda^{bb'}_{j_1j}(p)}{\mathrm{d}p^2}\frac{\partial^2}{\partial\boldsymbol{p}^2}C^{bb'}_{jj_2}(p)\right]. \tag{10}$$

In the diffusion limit we can also neglect the $B$-dependence of $\Lambda^{bb'}_{j_1j_2}(p)$, using the expression

$$\Lambda^{bb'}_{j_1j_2}(p) = \sum_{\boldsymbol{q}} G^{b}_{j_1j_2}(\boldsymbol{p}/2-\boldsymbol{q})G^{b'}_{j_1j_2}(\boldsymbol{p}/2+\boldsymbol{q}) \tag{11}$$

where $G^{b}_{j_1j_2}(\boldsymbol{p})$ are the matrix elements of the Green's function:

$$\hat{G}^{R,A}(\boldsymbol{p}) = \left[E_F - \frac{p^2}{2m} - \hat{h} \pm \hat{P}_l\frac{\mathrm{i}\hbar}{2}\left(\frac{1}{\tau_l}+\frac{1}{\tau_{\varphi l}}\right) \pm \hat{P}_r\frac{\mathrm{i}\hbar}{2}\left(\frac{1}{\tau_r}+\frac{1}{\tau_{\varphi r}}\right)\right]^{-1} \tag{12}$$

describing the double-layer system in the absence of the magnetic field. The Fermi energy is measured from the centre between the two levels of the double quantum well, the elastic scattering times are introduced as $\tau_j = \hbar^3/mw_j$, and the $\tau_{\varphi j}$ are the phenomenologically introduced inelastic scattering times.

In the weak-coupling limit we have

$$\frac{1}{\tau_t} = \frac{2(T/\hbar)^2\tau}{1+(\Delta\tau/\hbar)^2} \ll \frac{1}{\tau} \tag{13}$$

where $\tau = 2\tau_l\tau_r/(\tau_l+\tau_r)$ is the average elastic scattering time and $\tau_t$ the tunnelling time. A calculation of $\Lambda^{bb'}_{j_1j_2}(p)$ in the limit (13) gives

$$\Lambda^{RA}_{jj}(p) \simeq \frac{1}{w_j}\left(1 - \frac{\tau_j}{\tau_{\varphi j}} - D_j\tau_j\left(\frac{p}{\hbar}\right)^2 - \frac{\tau_j}{\tau_t}\right) \tag{14}$$

and

$$\Lambda^{RA}_{lr}(p) = \Lambda^{RA}_{rl}(p) \simeq \frac{\tau_l+\tau_r}{(w_l+w_r)\tau_t}. \tag{15}$$

In equation (14) we introduced the 2D diffusion coefficients in the layers $D_j = v^2_{Fj}\tau_j/2$. $\Lambda^{AR}_{jj'}(p)$ is given by equations (14) and (15) as well, while $\Lambda^{RR}_{jj'}(p)$ and $\Lambda^{AA}_{jj'}(p)$ are small and can be neglected. A substitution of equations (14) and (15) into equation (10) leads to the following equations:

$$-\frac{\hbar^2 D_l\tau_l}{\ell^4}\frac{\partial^2}{\partial\boldsymbol{p}^2}C^{RA}_{ll}(p) + \left(\frac{\tau_l}{\tau_{\varphi l}} + D_l\tau_l\left(\frac{p}{\hbar}\right)^2 + \frac{\tau_l}{\tau_t}\right)C^{RA}_{ll}(p) - \frac{\tau_r}{\tau_t}C^{RA}_{rl}(p) = w_l \tag{16}$$

$$-\frac{\hbar^2 D_r\tau_r}{\ell^4}\frac{\partial^2}{\partial\boldsymbol{p}^2}C^{RA}_{rl}(p) + \left(\frac{\tau_r}{\tau_{\varphi r}} + D_r\tau_r\left(\frac{p}{\hbar}\right)^2 + \frac{\tau_r}{\tau_t}\right)C^{RA}_{rl}(p) - \frac{\tau_l}{\tau_t}C^{RA}_{ll}(p) = 0. \tag{17}$$

$C^{RA}_{rr}(p)$ and $C^{RA}_{lr}(p)$ are given by equations (16) and (17) with the indices $l$ and $r$ interchanged.

Equations (16) and (17) can be diagonalized by the substitutions

$$C^{RA}_{ll}(p) = C^l_1(p) + C^l_2(p) \qquad \text{and} \qquad C^{RA}_{rl}(p) = \lambda_1 C^l_1(p) + \lambda_2 C^l_2(p).$$

For $C_k(p)$ ($k = 1, 2$) we obtain

$$\left[-\frac{\hbar^2}{\ell^4}\frac{\partial^2}{\partial\boldsymbol{p}^2} + \left(\frac{p}{\hbar}\right)^2 + q_k^2\right]C^l_k(p) = \frac{w_l}{\tau_l D_l}A_k \tag{18}$$

where

$$q^2_{1,2} = (s_l+s_r)/2 \pm S/2 \qquad A_{1,2} = 1/2 \pm (s_l-s_r)/2S. \tag{19}$$

Here $s_j = (D_j\tau_{\varphi j})^{-1} + (D_j\tau_t)^{-1}$, $s_t^2 = (D_lD_r\tau_t^2)^{-1}$, and $S = [(s_l-s_r)^2 + 4s_t^2]^{1/2}$. Equation (18) is analogous to that for the Green's function of a two-dimensional harmonic oscillator and its solution is obtained in a straightforward way. The result is

$$C^{RA}_{ll}(p) = 2\frac{w_l}{\tau_l D_l}\sum_n (-1)^n \mathrm{e}^{-u} L^0_n(2u)\left[\frac{A_1}{(2/\ell^2)(2n+1)+q_1^2} + \frac{A_2}{(2/\ell^2)(2n+1)+q_2^2}\right]. \tag{20}$$

The expression for $C_{rr}^{RA}(p)$ is given by equation (20) with the layer indices interchanged.

Both terms in equation (20) contain diffusion poles modified due to tunnelling. In the absence of tunnelling (uncoupled layers) we obtain the well-known result [4] for each layer. Now we calculate the weak-localization contribution $\delta\sigma_{\alpha\alpha'}$. In the diffusion limit, equation (4) gives

$$\delta\sigma_{\alpha\alpha'} = \frac{e^2\hbar}{2\pi m^2 L^2} \sum_{bb'=R,A} (-1)^l \sum_{jj'j_1j_2} \sum_{pp'} p_\alpha p'_{\alpha'} G^b_{jj_1}(\boldsymbol{p}) G^b_{j_2j'}(\boldsymbol{p}') \times G^{b'}_{j'j_1}(\boldsymbol{p}') G^{b'}_{j_2j}(\boldsymbol{p}) C^{bb'}_{j_1j_2}(|\boldsymbol{p}+\boldsymbol{p}'|). \tag{21}$$

In the limit (13) we should retain only the terms with $j = j' = j_1 = j_2$. The weak-localization contribution is expressed through $C_{ll}^{RA}(p)$ and $C_{rr}^{RA}(p)$. It gives a positive magnetoconductivity $\Delta\sigma = \delta\sigma(B) - \delta\sigma(0)$ as a sum of two parts [9]:

$$\frac{\Delta\sigma}{\delta\sigma_0} = f\left(\frac{4}{\ell^2 q_1^2}\right) + f\left(\frac{4}{\ell^2 q_2^2}\right). \tag{22}$$

Here $\delta\sigma_0 = e^2/2\pi^2\hbar$, and $f(x) = \psi(1/2 + 1/x) + \ln x$, where $\psi(x)$ is the digamma function. In the absence of the magnetic field, the weak-localization contribution, within logarithmic accuracy, is given as

$$\delta\sigma(0) = \delta\sigma_0 \ln\left(\frac{\tau_l\tau_r}{\tau_{\varphi l}\tau_{\varphi r}} + 2\,\frac{\tau_l\tau_r}{\tau_t\tau_\varphi}\right) \tag{23}$$

where we have introduced an average inelastic scattering time $\tau_\varphi = 2\tau_{\varphi l}\tau_{\varphi r}/(\tau_{\varphi l} + \tau_{\varphi r})$. It is convenient to analyse the results (22) and (23) in the symmetric case described by $D_l = D_r = D$, $\tau_l = \tau_r$, and $\tau_{\varphi l} = \tau_{\varphi r}$. In this case equation (23) takes the form $\delta\sigma(0) = \delta\sigma_0[\ln(\tau/\tau_\varphi) + \ln(\tau/\tau_\varphi + 2\tau/\tau_t)]$. The first term is the same as in the absence of tunnelling while the second one is tunnelling dependent. In the symmetric case, equation (22) gives

$$\frac{\Delta\sigma}{\delta\sigma_0} = f\left(\frac{B}{B_0}\right) + f\left(\frac{B}{B_0(1 + 2\tau_\varphi/\tau_t)}\right) \tag{24}$$

where $B_0 = \hbar/4eD\tau_\varphi$ is the characteristic field. In weak magnetic fields, the magnetoconductivity increases according to a $B^2$-law. In stronger fields, when the magnetic length $\ell$ becomes less than $\sqrt{D\tau_t}$, $\delta\sigma(B)$ loses its dependence on the tunnelling and the system behaves like two decoupled layers. In the limit $B \gg B_0$, $B \gg B_0(\tau_\varphi/\tau_t)$ the magnetoconductivity is proportional to $2\,\delta\sigma_0 \ln B$. For $\tau_t \ll \tau_\varphi$, an intermediate regime exists, $B_0 \ll B \ll B_0(\tau_\varphi/\tau_t)$, in which the magnetoconductivity is proportional to $\delta\sigma_0 \ln(B)$. Figure 1 shows the magnetic field dependence of the magnetoconductivity described by equation (24) for several values of $\tau_\varphi/\tau_t$.

## 4. Parallel magnetic field

Due to the spatial separation of the layers, a field $B$ applied parallel to them can considerably influence the conductivity of the double-layer system; see, for example, references [19–22]. This effect occurs as a result of the modification of the electron energy spectrum of the coupled layers by the field $B$. However, under the condition $\tau_t \gg \tau$ the coherent tunnel coupling is suppressed by the elastic scattering and the effect disappears. In contrast, the influence of this in-plane field $B$ on the weak-localization part of the conductivity should exist as long as $\tau_t < \tau_\varphi$; the corresponding characteristic field $B_1$ has to be much smaller. An assessment of this influence is given below in the limit (13).

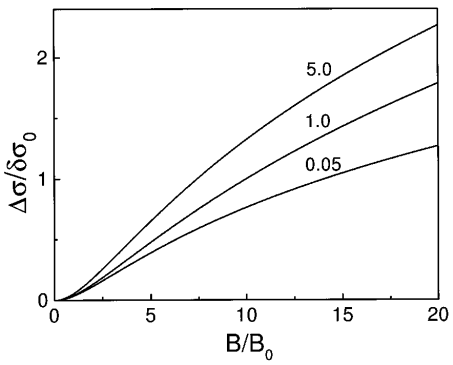


**Figure 1.** Magnetoconductivity of the double-layer system in a *perpendicular* magnetic field $B$ for symmetric conditions. The curves are marked with the values of the ratio $\tau_t/\tau_\varphi$ and $B_0 = \hbar/4eD\tau_\varphi$.

The parallel field renders the electron Green's function anisotropic. For $\boldsymbol{B} = (0, B, 0)$ we choose $\boldsymbol{A} = (B(z - z_0), 0, 0)$ with $z_0 = (\langle l|\, z\, |l\rangle + \langle r|\, z\, |r\rangle)/2$ and obtain [20, 21]

$$\hat{G}^{R,A}(\boldsymbol{p}) = \left[E_F - \frac{p^2}{2m} - \frac{\Delta_{p_x}}{2}\hat{\sigma}_z - T\hat{\sigma}_x \pm \hat{P}_l \frac{\mathrm{i}\hbar}{2}\left(\frac{1}{\tau_l} + \frac{1}{\tau_{\varphi l}}\right) \pm \hat{P}_r \frac{\mathrm{i}\hbar}{2}\left(\frac{1}{\tau_r} + \frac{1}{\tau_{\varphi r}}\right)\right]^{-1} \tag{25}$$

where $\Delta_{p_x} = \Delta - \omega_c Z p_x + \delta_B$, $\omega_c = eB/m$ is the cyclotron frequency, $Z = \langle r|\, z\, |r\rangle - \langle l|\, z\, |l\rangle$ is the interlayer distance, and $\delta_B \sim B^2$ is a small field-dependent correction to the level splitting which can be neglected for small $B$.

Using equation (25) we can calculate $\Lambda^{bb'}_{jj'}(\boldsymbol{p})$ from equation (11). In contrast to the case for the previous section, $\Lambda^{bb'}_{jj'}(\boldsymbol{p})$ is now anisotropic. In the limit $\ell^2 \gg Z v_{Fj} \tau_j$, instead of equation (14) we have

$$\Lambda^{RA}_{jj}(\boldsymbol{p}) \simeq \frac{1}{w_j}\left(1 - \frac{\tau_j}{\tau_{\varphi j}} - D_j \tau_j [(p_x \mp p_B)^2 + p_y^2]/\hbar^2 - \frac{\tau_j}{\tau_t}\right) \tag{26}$$

where $p_B = m\omega_c Z = eBZ$, and the upper (lower) sign stands for $j = l$ ($j = r$). Equation (15) remains unchanged. The Bethe–Salpeter equation for the Cooperon now reduces to a set of linear algebraic equations:

$$C^{bb'}_{j_1 j_2}(\boldsymbol{p}) = w_{j_1} \Lambda^{bb'}_{j_1 j_2}(\boldsymbol{p}) w_{j_2} + w_{j_1} \sum_j \Lambda^{bb'}_{j_1 j}(\boldsymbol{p}) C^{bb'}_{j j_2}(\boldsymbol{p}) \tag{27}$$

whose solution is straightforward. A calculation of $\delta\sigma_{\alpha\alpha'}$ can be done according to equation (21), with $C^{bb'}_{j_1 j_2}(|\boldsymbol{p} + \boldsymbol{p}'|)$ replaced by $C^{bb'}_{j_1 j_2}(\boldsymbol{p} + \boldsymbol{p}')$ found from equation (27). Since $\tau_t \gg \tau$ holds, we again put $j = j' = j_1 = j_2$ and finally obtain

$$\Delta\sigma = \Delta\sigma_{sat} - \delta\sigma_0 \sum_{j=l,r} \int_0^\infty \mathrm{d}x \, \frac{s_t^2}{(x + s_j)\sqrt{R_j(x)}} \tag{28}$$

$$\Delta\sigma_{sat} = \delta\sigma_0 \ln\left[\left(1 + \frac{\tau_{\varphi l}}{\tau_t}\right)\left(1 + \frac{\tau_{\varphi r}}{\tau_t}\right) \Big/ \left(1 + \frac{\tau_{\varphi l} + \tau_{\varphi r}}{\tau_t}\right)\right] \tag{29}$$

where $x = (p/\hbar)^2$ and $R_j(x)$ are the fourth-order polynomials defined by

$$\begin{aligned} R_l(x) = {} & x^4 + 2(s_l + s_r - x_B)x^3 + [(s_l + s_r)^2 + 2(s_l s_r - s_t^2) + 2x_B(s_r - 2s_l) + x_B^2]x^2 \\ & + 2[(s_l + s_r)(s_l s_r - s_t^2) + x_B(2s_l s_r - s_l^2 - s_t^2) + x_B^2 s_l]x + [s_l s_r - s_t^2 + x_B s_l]^2 \end{aligned} \tag{30}$$

with $x_B = (2p_B/\hbar)^2$. The result for $R_r(x)$ is given by equation (30) with the layer indices interchanged. Although the field $B$ induces an anisotropy in the Green's functions, the localization correction $\delta\sigma(B)$ is isotropic because the main contribution in equation (21) comes from $|\boldsymbol{p}| \simeq |\boldsymbol{p}'| \simeq mv_{Fj}$.

Equations (28)–(30) show that in the weak-field region $B \ll B_1$, where $B_1 = \hbar/eZ\sqrt{D\tau_\varphi}$ and $D = 2D_lD_r/(D_l + D_r)$ is the average diffusion coefficient, the magnetoconductivity follows a $B^2$-law:

$$\frac{\Delta\sigma}{\delta\sigma_0} = \left(\frac{2s_teZB}{\hbar S}\right)^2 \left[\frac{s_l + s_r}{s_ls_r - s_t^2} - \frac{2}{S}\ln\left(\frac{s_l + s_r + S}{s_l + s_r - S}\right)\right]. \tag{31}$$

At $\tau_t \ll \tau_{\varphi j}$ we obtain $\Delta\sigma/\delta\sigma_0 = (B/B_1)^2$. In the opposite limit, a simple expression for the magnetoconductivity can be obtained in the symmetric case: $\Delta\sigma/\delta\sigma_0 = \frac{4}{3}(\tau_\varphi/\tau_t)^2(B/B_1)^2$; it shows a substantial suppression of the magnetoconductivity as a result of decreasing tunnelling probability.

With the increase of $B$, the $B$-dependence becomes weaker. For $B \gg B_1$ and $B^2 \gg B_1^2\tau_\varphi/\tau_t$, the integral in equation (28) can be neglected and the magnetoconductivity is saturated and equal to $\Delta\sigma_{sat}$ defined by equation (29). For $\tau_\varphi/\tau_t \ll 1$ the saturated value goes to zero. Recently, a saturation behaviour of the weak-localization contribution in parallel magnetic fields has been theoretically found for superlattices [15].

When the condition $\tau_t \ll \tau_\varphi$ holds, one can analytically evaluate the magnetoconductivity for $B^2 \ll B_1^2\tau_\varphi/\tau_t$; the result is

$$\Delta\sigma = \delta\sigma_0 \ln[1 + (B/B_1)^2]. \tag{32}$$

It shows that in the intermediate region $B_1^2 \ll B^2 \ll B_1^2\tau_\varphi/\tau_t$, the magnetoconductivity follows a logarithmic law. In figure 2 we demonstrate a transition from a $B^2$-behaviour at weak fields to the saturation at stronger fields for several values of the ratio $\tau_t/\tau_\varphi$.

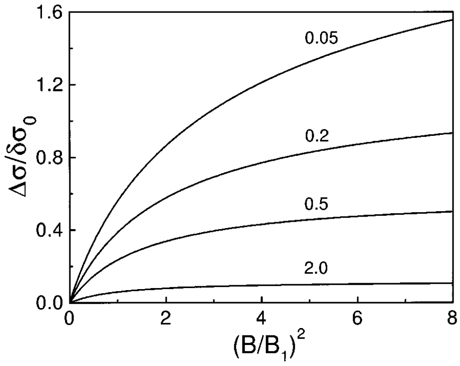


**Figure 2.** Magnetoconductivity of the double-layer system in a *parallel* magnetic field $B$ for symmetric conditions. The curves are marked as in figure 1 and $B_1 = \hbar/eZ\sqrt{D\tau_\varphi}$

The tunnelling rate $1/\tau_t$ can be varied by applying a transverse voltage which changes the level splitting $\Delta$ [18]. In the case under consideration $E_F \gg \hbar/\tau$, a relatively small variation of $\Delta$ has no great influence on the electron densities in the wells, but can dramatically modify the tunnelling rate. Figure 3 shows the $\Delta$-dependence of $\delta\sigma(B) - \delta\sigma(0)|_{\Delta=0}$ for several values of the magnetic field, for both *parallel* (solid) and *perpendicular* (dashed lines) fields; the thick solid line is the result for $B = 0$. The curves have a typical resonance-like behaviour that is affected differently by the two field orientations. An increase of $\Delta$ always leads to a

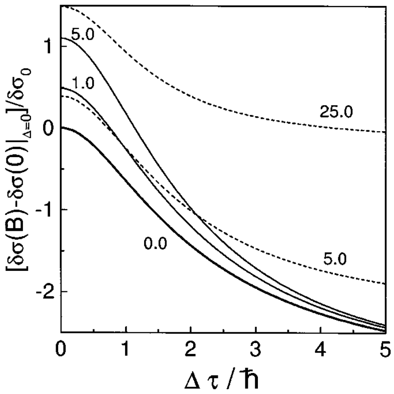


**Figure 3.** The relative weak-localization correction to the conductivity of the double-layer system as a function of the level splitting $\Delta$ for several values of the *parallel* (solid) and *perpendicular* (dashed) magnetic field. It is assumed that $\tau_t/\tau_\varphi = 0.1$ at $\Delta = 0$. The numbers next to the curves show the values of $B/B_0$ (perpendicular field) and $(B/B_1)^2$ (parallel field).

decrease of the conductivity, because in this way the tunnelling is suppressed and, therefore, the interference is increased. The perpendicular magnetic field tends to smooth this effect because it suppresses the dependence of the weak-localization part of the conductivity on the tunnelling; see section 3. On the other hand, the parallel field tends to strengthen the tunnelling dependence of the weak-localization part: in the saturation regime, cf. figure 2, this dependence is the strongest.

Above, we calculated the magnetoconductivity in the weak-tunnelling regime. Below, we evaluate it again in the regime of coherent tunnel coupling.

## 5. Coherent tunnelling regime

The aim of this section is to study the weak-localization effects at sufficiently strong, coherent tunnel coupling, described by $\tau_t \sim \tau$ or even $\tau_t \ll \tau$. In these conditions one always has $\tau_t \ll \tau_\varphi$, which means that there is no competition between the tunnelling and inelastic scattering processes. As a result, instead of two diffusion poles—see equation (20)—the expression for the Cooperon contains just one pole as for a single-layer system. However, the properties of this pole are somewhat different to those in the case of a single 2D layer and require a separate consideration.

Below, we assume that $E_F$ is large in comparison with $T$ and $\Delta$. Introducing a single Fermi velocity $v_F$ for both layers, we write the average diffusion coefficient as $D = v_F^2\tau/2$. The calculation of the Cooperon according to equations (11), (25), and (27) gives

$$C_{jj'}(\boldsymbol{p}) = \frac{w_j w_{j'}}{w_l + w_r}\left[\frac{\tau}{\tau_\varphi} + \frac{D(\Delta)\tau}{\hbar^2}(p^2 + r(\Delta)p_B^2 + 2\mu r(\Delta)p_B p_x)\right]^{-1}. \quad (33)$$

The inelastic scattering time enters the diffusion pole only as an average. On the other hand, the Cooperon depends on the asymmetry of the elastic scattering described by the parameter $\mu = (\tau_r - \tau_l)/(\tau_r + \tau_l)$. In equation (33) we have also introduced a $\Delta$-dependent diffusion coefficient according to

$$D(\Delta) = \frac{D}{1 - \mu^2 r(\Delta)} \qquad r(\Delta) = \frac{\Delta^2 + (\hbar/\tau)^2}{\Delta^2 + 4T^2 + (\hbar/\tau)^2}. \quad (34)$$

Note that $D(\Delta)$ differs from $D$ only if the elastic scattering is asymmetric.

If there is no parallel magnetic field ($p_B = 0$), equation (33) has the same form as the Cooperon of a single-layer system. Therefore, in the diffusion regime, equation (33) with the substitution $p^2 \to 4\hbar eB(n + 1/2)$ can be directly used to calculate the magnetoconductivity in a perpendicular magnetic field $B$. Using equation (21), we obtain, in analogy with the single-layer case,

$$\Delta\sigma = \delta\sigma_0\, f\left(\frac{4eD(\Delta)B\tau_\varphi}{\hbar}\right) \tag{35}$$

and, with logarithmic accuracy, $\delta\sigma(0) = \delta\sigma_0 \ln(\tau/\tau_\varphi)$. The only difference between equation (35) and the well-known expression for a single-layer 2D system is the replacement of $D$ in the latter by $D(\Delta)$. The physical meaning of this change is clear and easily understood in terms of the resistance resonance phenomenon [23, 24]; see also reference [18]. In the corresponding theory [23, 24], the factor $1/[1 - \mu^2 r(\Delta)]$ describes the $\Delta$-dependence of the conductivity and of the diffusion coefficient $D$ of the double quantum wells with asymmetric scattering. With the increase of $|\Delta|$, $D$ increases and leads to an increase of the magnetoconductivity $\Delta\sigma$.

Now we consider the case of a parallel magnetic field. Using equations (33) and (21), we obtain

$$\Delta\sigma = \delta\sigma_0 \ln\left[1 + r(\Delta)(B/B_1)^2\right]. \tag{36}$$

The magnetoconductivity described by equation (36) does not depend on the asymmetry of the scattering. On the other hand, its dependence on $\Delta$ has the same sign as in the case of the perpendicular field: when $\Delta$ increases, $r(\Delta)$ goes to unity and $\Delta\sigma$ increases. The behaviour of the magnetoconductivity described by equations (35) and (36) is illustrated in figure 4.

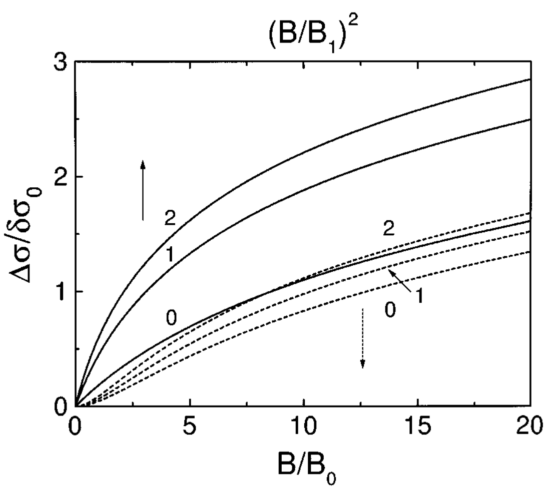


**Figure 4.** Magnetoconductivity of the double-layer system in a coherent tunnelling regime for $2T\tau/\hbar = 4$ and $\mu = 0.7$. The solid and dashed lines correspond to parallel and perpendicular fields, respectively. The curves are marked with the values of the ratio $\Delta/2T$.

The results for the magnetoconductivity described in this section are valid for $\tau_t \ll \tau_\varphi$ under the conditions $D\tau_t eB/\hbar \ll 1$ for perpendicular field $B$ and $D\tau_t(eZB/\hbar)^2 \ll 1$ for parallel field $B$. In the limit (13), when $r(\Delta) = 1$, equation (35) is equivalent to equation (22) written at $\tau_t \ll \tau_\varphi$ and $D\tau_t eB/\hbar \ll 1$, while equation (36) is equivalent to equation (32).

## 6. Discussion

We have investigated the influence of the tunnel coupling between two 2D layers on the weak-localization-induced magnetoconductivity. This coupling introduces an extra degree of freedom for an electron giving it the possibility of tunnelling between the layers and, therefore, it reduces the interference effects. As a result, the weak-localization contribution is reduced as the tunnelling rate $1/\tau_t$ gradually prevails over the inelastic scattering rate. In particular, it leads to the weakening of the magnetoresistance in the perpendicular magnetic field; see section 3.

The results described in section 3 are similar to those obtained in reference [9] for two thin metallic films separated by a barrier if one makes $\tau_t$ equal to the tunnelling times $\tau_{12}$ and $\tau_{21}$ of reference [9]. This is not surprising. When the inelastic scattering lengths are small in comparison to the film widths, the diffusion of the electrons in such a system proceeds in the same way as in coupled 2D layers. A characteristic feature of the double-quantum-well case, as opposed to the case of thin films, is the strong resonance dependence of the tunnelling time $\tau_t$, cf. equation (13), on the energy-level splitting $\Delta$. The latter can be easily controlled by external gates enclosing the double quantum wells. This opens up a new way to examine the weak-localization phenomena by measuring the $\Delta$-dependence of the conductivity—see, for example, figure 3—in addition to the $B$-dependence. The conductivity, due to its weak-localization part, is maximal when the tunnelling resonance condition $\Delta = 0$ holds (this can be called 'delocalization resonance'), while an increase of $\Delta$ increases the interference and leads to a decrease of the conductivity down to its value for uncoupled layers. On the other hand, if the scattering times in the layers are different, the main part of the conductivity will show the opposite effect (the resistance resonance [18]). Although the relative size of the resistance resonance would be small due to the parameter $\tau/\tau_t$, it can obscure the delocalization resonance. Therefore, the samples with symmetric scattering are preferable for probing the interference corrections.

In section 4 we studied the magnetoconductivity in a parallel magnetic field. Since this magnetoconductivity can exist only in the presence of the tunnel coupling, it becomes more pronounced as the tunnelling rate prevails over the inelastic scattering rate. Although at very small $B$ both parallel and perpendicular fields give $B^2$-corrections to the magnetoconductivity, at higher fields the behaviour becomes completely different. When the *perpendicular* field is larger than both $B_0$ and $B_0\tau_\varphi/\tau_t$, the weak-localization contribution increases logarithmically with $B$ and does not depend on the tunnelling. When the *parallel* field is larger than both $B_1$ and $B_1\sqrt{\tau_\varphi/\tau_t}$, $\delta\sigma(B)$ becomes saturated, i.e., it loses its dependence on $B$ but not that on the tunnelling. Since $(v_F/Z)\sqrt{\tau_\varphi\tau} \gg 1$ holds, the characteristic parallel field $B_1$ is much larger than the characteristic perpendicular field $B_0$.

With an increase of $\Delta$, both $\delta\sigma(B)$ and $\Delta\sigma$ in a parallel magnetic field decrease. However, in the case of coherent tunnel coupling (section 5) the behaviour of the magnetoconductivity $\Delta\sigma$ versus $\Delta$ is opposite: an increase of $\Delta$ increases $\Delta\sigma$. A transition between these two kinds of behaviour can be explained as follows. When the tunnel coupling becomes so strong that $D\tau_t(eBZ/\hbar)^2 \ll 1$, a change of $\tau_t$ with $\Delta$ no longer modifies the magnetic field suppression of the interference; see section 4. With a further increase of the tunnelling rate, when the latter prevails over the elastic scattering rate, coherent coupling effects become important. Instead of the states localized in the wells, one has a pair of hybridized states, which are well defined in the limit of $2T \gg \hbar/\tau$. At $\Delta = 0$ the electron density in both of these states is distributed equally among the wells, and the parallel magnetic field cannot reduce the interference in the way described in the introduction. The magnetoconductivity is zero, which is given also by equation (36) at $2T \gg \hbar/\tau$ and $\Delta = 0$. At $\Delta \neq 0$ the symmetry is broken. The two states are mostly localized in different wells and the parallel magnetic field can reduce the interference because transitions of electrons between these states are possible. In conclusion, the magnetoconductivity increases with an increase of $\Delta$.

Let us briefly discuss the main approximations used in this paper. Since we neglected the electron–electron interaction, which also leads to quantum corrections to the conductivity, the theory is applicable at temperatures considerably lower than $\hbar/\tau_\varphi$ [25]. We also neglected the possibility of spin-flip scattering, assuming that the relevant scattering time is larger than $\tau_\varphi$. The assumption of $\delta$-correlated scattering does not lead to principal changes. For example, if one takes into account the interlayer correlation of the scattering amplitudes, one has only to replace $1/\tau$ in equation (13) by $1/\tau - 1/\tau_{lr}$, where $\tau_{lr} = \hbar^3/mw_{lr}$.

Throughout the paper we calculated the total, i.e. summed over both layers, in-plane conductivity of the double-layer system. Experimentally, it corresponds to double quantum wells with common contacts to the layers. The case of independently contacted layers does not require a separate consideration for the following reason. Since the system is assumed macroscopic, its in-plane size should substantially exceed the inelastic scattering length $v_F\tau_\varphi$. On the other hand, the tunnel coupling manifests itself in weak-localization phenomena under the condition $\tau_\varphi > \tau_t$. In such a case, a characteristic current redistribution length [26]

$$l_s \sim \sqrt{v_F^2\tau\tau_t}$$

is small in comparison to the size of the system, which means that for any case of independent contacting, the double-layer system would behave as a system with common contacts.

Finally, since we analysed in detail the results for parallel and perpendicular fields $B$, our study could be of help in describing weak localization in tilted magnetic fields by decomposing $B$ into the relevant components. For a compact treatment that encompasses both orientations, applicable to *single* quasi-2D or 3D systems and/or superlattices but not to double wells as investigated here, we refer the reader to reference [27].

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